

Rioplatense Mathematical Olympiad, Level 3 2008

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by Shu

Day 1

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- 1** In each square of a chessboard with a rows and b columns, a 0 or 1 is written satisfying the following conditions.
- If a row and a column intersect in a square with a 0, then that row and column have the same number of 0s.
 - If a row and a column intersect in a square with a 1, then that row and column have the same number of 1s.
- Find all pairs (a, b) for which this is possible.
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- 2** On a line, there are n closed intervals (none of which is a single point) whose union we denote by S . It's known that for every real number d , $0 < d \leq 1$, there are two points in S that are a distance d from each other.
- (a) Show that the sum of the lengths of the n closed intervals is larger than $\frac{1}{n}$.
- (b) Prove that, for each positive integer n , the $\frac{1}{n}$ in the statement of part (a) cannot be replaced with a larger number.
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- 3** Find all integers $k \geq 2$ such that for all integers $n \geq 2$, n does not divide the greatest odd divisor of $k^n + 1$.
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Day 2

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- 1** Can the positive integers be partitioned into 12 subsets such that for each positive integer k , the numbers $k, 2k, \dots, 12k$ belong to different subsets?
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- 2** In triangle ABC , where $AB < AC$, let X, Y, Z denote the points where the incircle is tangent to BC, CA, AB , respectively. On the circumcircle of ABC , let U denote the midpoint of the arc BC that contains the point A . The line UX meets the circumcircle again at the point K . Let T denote the point of intersection of AK and YZ . Prove that XT is perpendicular to YZ .
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- 3** Consider a collection of stones whose total weight is 65 pounds and each of whose stones is at most w pounds. Find the largest number w for which any such collection of stones can be divided into two groups whose total weights differ by at most one pound.
- Note: The weights of the stones are not necessarily integers.
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