Turkey Junior National Olympiad 1998

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1 Let F, D, and E be points on the sides $[AB]$, $[BC]$, and $[CA]$ of $\triangle ABC$, respectively, such that $\triangle DEF$ is an isosceles right triangle with hypotenuse $[EF]$. The altitude of $\triangle ABC$ passing through A is 10 cm. If $|BC| = 30$ cm, and $EF \parallel BC$, calculate the perimeter of $\triangle DEF$.

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2 The first 9 positive integers are placed into the squares of a 3×3 chessboard. We are taking the smallest number in a column. Let a be the largest of these three smallest number. Similarly, we are taking the largest number in a row. Let b be the smallest of these three largest number. How many ways can we distribute the numbers into the chessboard such that $a = b = 4$?

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3 We call a positive integer *good number*, if it is divisible by squares of all its prime factors. Show that there are infinitely many pairs of consequtive numbers both are *good*.
