Turkey Junior National Olympiad 2000

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1 Let ABC be a triangle with $\angle BAC = 90^\circ$. Construct the square $BDEC$ such as A and the square are at opposite sides of BC. Let the angle bisector of $\angle BAC$ cut the sides $[BC]$ and $[DE]$ at F and G, respectively. If $|AB| = 24$ and $|AC| = 10$, calculate the area of quadrilateral $BDGF$.

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2 Find the least positive integer n such that 15 divides the product

$$a_1 a_2 \dots a_{15} \left(a_1^n + a_2^n + \dots + a_{15}^n\right)$$

, for every positive integers $a_1, a_2, \dots, a_{15}$.

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3 $f : \mathbb{R} \to \mathbb{R}$ satisfies the equation

$$f(x)f(y) - af(xy) = x + y$$

, for every real numbers x, y. Find all possible real values of a.
