Turkey Junior National Olympiad 2005

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1   Let ABC be an acute triangle. Let H and D be points on $[AC]$ and $[BC]$, respectively, such that $BH \perp AC$ and $HD \perp BC$. Let O_1 be the circumcenter of $\triangle ABH$, and O_2 be the circumcenter of $\triangle BHD$, and O_3 be the circumcenter of $\triangle HDC$. Find the ratio of area of $\triangle O_1O_2O_3$ and $\triangle ABH$.

2   Find all integer pairs (x, y) such that $x^3 + y^3 = (x + y)^2$.

3   Determine whether or not there exists a sequence of integers $a_1, a_2, \ldots, a_{19}, a_{20}$ such that, the sum of all the terms is negative, and the sum of any three of the terms is positive.