Turkey Junior National Olympiad 2006

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1   Let $ABCD$ be a trapezoid such that $AD \parallel BC$. The interior angle bisectors of the corners A and B meet on $[DC]$. If $|BC| = 9$ and $|AD| = 4$, find $|AB|$.

2   Find all integer triples (x, y, z) such that

$$\begin{aligned} x - yz &= 11 \\ xz + y &= 13. \end{aligned}$$

3   In the beginnig, all nine squares of 3×3 chessboard contain 0. At each step, we choose two squares sharing a common edge, then we add 1 to them or -1 to them. Show that it is not possible to make all squares 2, after a finite number of steps.