Turkey Junior National Olympiad 2010

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1   A circle that passes through the vertex A of a rectangle $ABCD$ intersects the side AB at a second point E different from B. A line passing through B is tangent to this circle at a point T, and the circle with center B and passing through T intersects the side BC at the point F. Show that if $\angle CDF = \angle BFE$, then $\angle EDF = \angle CDF$.

2   Determine the number of positive integers n for which $(n+15)(n+2010)$ is a perfect square.

3   In an exam every question is solved by exactly four students, every pair of questions is solved by exactly one student, and none of the students solved all of the questions. Find the maximum possible number of questions in this exam.

4   Prove that
$$a^2b^2(a^2 + b^2 - 2) \geq (a+b)(ab-1)$$
for all positive real numbers a and b.