

Turkey Junior National Olympiad 2011

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by xeroxia

- 1 Show that

$$1 \leq \frac{(x+y)(x^3+y^3)}{(x^2+y^2)^2} \leq \frac{9}{8}$$

holds for all positive real numbers x, y .

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- 2 Let ABC be a triangle with $|AB| = |AC|$. D is the midpoint of $[BC]$. E is the foot of the altitude from D to AC . BE cuts the circumcircle of triangle ABD at B and F . DE and AF meet at G . Prove that $|DG| = |GE|$

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- 3 $m < n$ are positive integers. Let $p = \frac{n^2+m^2}{\sqrt{n^2-m^2}}$.

(a) Find three pairs of positive integers (m, n) that make p prime.

(b) If p is prime, then show that $p \equiv 1 \pmod{8}$.

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- 4 Each student chooses 1 math problem and 1 physics problem among 20 math problems and 11 physics problems. No same pair of problem is selected by two students. And at least one of the problems selected by any student is selected by at most one other student. At most how many students are there?
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