Turkey Junior National Olympiad 2013

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by crazyfehmy

1 Let x, y, z be real numbers satisfying $x+y+z=0$ and $x^2+y^2+z^2=6$. Find the maximum value of

$$|(x-y)(y-z)(z-x)|$$

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2 Find all prime numbers p, q, r satisfying the equation

$$p^4+2p+q^4+q^2=r^2+4q^3+1$$

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3 Let ABC be a triangle such that $AC > AB$. A circle tangent to the sides AB and AC at D and E respectively, intersects the circumcircle of ABC at K and L. Let X and Y be points on the sides AB and AC respectively, satisfying

$$\frac{AX}{AB}=\frac{CE}{BD+CE} \quad \text{and} \quad \frac{AY}{AC}=\frac{BD}{BD+CE}$$

Show that the lines XY, BC and KL are concurrent.

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4 Player A places an odd number of boxes around a circle and distributes 2013 balls into some of these boxes. Then the player B chooses one of these boxes and takes the balls in it. After that the player A chooses half of the remaining boxes such that none of two are consecutive and take the balls in them. If player A guarantees to take k balls, find the maximum possible value of k.
