Turkey Junior National Olympiad 2014

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by bcp123

1 Prove that for positive reals a,b,c so that $a+b+c+abc=4$,

$$\left(1+\frac{a}{b}+ca\right)\left(1+\frac{b}{c}+ab\right)\left(1+\frac{c}{a}+bc\right)\geq 27$$

holds.

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2 Determine the minimum possible amount of distinct prime divisors of $19^{4n}+4$, for a positive integer n.

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3 There are 2014 balls with 106 different colors, 19 of each color. Determine the least possible value of n so that no matter how these balls are arranged around a circle, one can choose n consecutive balls so that amongst them, there are 53 balls with different colors.

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4 ABC is an acute triangle with orthocenter H. Points D and E lie on segment BC. Circumcircle of $\triangle BHC$ instersects with segments AD,AE at P and Q, respectively. Prove that if $BD^2+CD^2=2DP\cdot DA$ and $BE^2+CE^2=2EQ\cdot EA$, then $BP=CQ$.
