Mexico National Olympiad 2007

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Day 1

1 Find all integers N with the following property: for 10 but not 11 consecutive positive integers, each one is a divisor of N.

2 Given an equilateral $\triangle ABC$, find the locus of points P such that $\angle APB=\angle BPC$.

3 Given a, b, and c be positive real numbers with $a+b+c=1$, prove that

$$\sqrt{a+bc}+\sqrt{b+ca}+\sqrt{c+ab}\le 2$$

Day 2

1 The fraction $\frac{1}{10}$ can be expressed as the sum of two unit fraction in many ways, for example, $\frac{1}{30}+\frac{1}{15}$ and $\frac{1}{60}+\frac{1}{12}$.

Find the number of ways that $\frac{1}{2007}$ can be expressed as the sum of two distinct positive unit fractions.

2 In each square of a 6×6 grid there is a lightning bug on or off. One move is to choose three consecutive squares, either horizontal or vertical, and change the lightning bugs in those 3 squares from off to on or from on to off. Show if at the beginning there is one lighting bug on and the rest of them off, it is not possible to make some moves so that at the end they are all turned off.

3 Let ABC be a triangle with $AB>BC>CA$. Let D be a point on AB such that $CD=BC$, and let M be the midpoint of AC. Show that $BD=AC$ and that $\angle BAC=2\angle ABM$.