**Czech-Polish-Slovak Match 2002**

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by djb86

**Day 1** June 17th

**1** Let $a, b$ be distinct real numbers and $k, m$ be positive integers $k+m=n \geq 3, k \leq 2m, m \leq 2k$. Consider sequences $x_1, \ldots, x_n$ with the following properties:
(i) $k$ terms $x_i$, including $x_1$, are equal to $a$;
(ii) $m$ terms $x_i$, including $x_n$, are equal to $b$;
(iii) no three consecutive terms are equal.
Find all possible values of $x_n x_1 x_2 + x_1 x_2 x_3 + \cdots + x_{n-1} x_n x_1$.

**2** A triangle $ABC$ has sides $BC=a, CA=b, AB=c$ with $a<b<c$ and area $S$. Determine the largest number $u$ and the least number $v$ such that, for every point $P$ inside $\triangle ABC$, the inequality $u \leq PD+PE+PF \leq v$ holds, where $D, E, F$ are the intersection points of $AP, BP, CP$ with the opposite sides.

**3** Let $S=\{1,2,\cdots,n\}, n \in N$. Find the number of functions $f: S \rightarrow S$ with the property that $x+f(f(f(f(x))))=n+1$ for all $x \in S$?

**Day 2** June 18th

**4** An integer $n>1$ and a prime $p$ are such that $n$ divides $p-1$, and $p$ divides $n^3-1$. Prove that $4p-3$ is a perfect square.

**5** In an acute-angled triangle $ABC$ with circumcenter $O$, points $P$ and $Q$ are taken on sides $AC$ and $BC$ respectively such that $\frac{AP}{PQ}=\frac{BC}{AB}$ and $\frac{BQ}{PQ}=\frac{AC}{AB}$ . Prove that the points $O, P, Q, C$ lie on a circle.

**6** Let $n \geq 2$ be a fixed even integer. We consider polynomials of the form

$$P(x)=x^n+a_{n-1}x^{n-1}+\cdots+a_1x+1$$

with real coefficients, having at least one real roots. Find the least possible value of $a_1^2+a_2^2+\cdots+a_{n-1}^2$.