Czech-Polish-Slovak Match 2006

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Day 1 June 26th

1 Five distinct points A, B, C, D and E lie in this order on a circle of radius r and satisfy $AC = BD = CE = r$. Prove that the orthocentres of the triangles ACD, BCD and BCE are the vertices of a right-angled triangle.

2 There are n children around a round table. Erika is the oldest among them and she has n candies, while no other child has any candy. Erika decided to distribute the candies according to the following rules. In every round, she chooses a child with at least two candies and the chosen child sends a candy to each of his/her two neighbors. (So in the first round Erika must choose herself). For which $n \geq 3$ is it possible to end the distribution after a finite number of rounds with every child having exactly one candy?

3 The sum of four real numbers is 9 and the sum of their squares is 21. Prove that these numbers can be denoted by a, b, c, d so that $ab - cd \geq 2$ holds.

Day 2 June 27th

4 Show that for every integer $k \geq 1$ there is a positive integer n such that the decimal representation of 2^n contains a block of exactly k zeros, i.e. $2^n = \dots a00\dots0b\cdots$ with k zeros and $a, b \neq 0$.

5 Find the number of sequences $(a_n)_{n=1}^{\infty}$ of integers satisfying $a_n \neq -1$ and

$$a_{n+2} = \frac{a_n + 2006}{a_{n+1} + 1}$$

for each $n \in \mathbb{N}$.

6 Find out if there is a convex pentagon $A_1A_2A_3A_4A_5$ such that, for each $i = 1, \dots, 5$, the lines A_iA_{i+3} and $A_{i+1}A_{i+2}$ intersect at a point B_i and the points B_1, B_2, B_3, B_4, B_5 are collinear. (Here $A_{i+5} = A_i$.)

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