

Junior Balkan MO 1997

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- 1 Show that given any 9 points inside a square of side 1 we can always find 3 which form a triangle with area less than $\frac{1}{8}$.

Bulgaria

- 2 Let $\frac{x^2+y^2}{x^2-y^2} + \frac{x^2-y^2}{x^2+y^2} = k$. Compute the following expression in terms of k :

$$E(x, y) = \frac{x^8 + y^8}{x^8 - y^8} - \frac{x^8 - y^8}{x^8 + y^8}.$$

Ciprus

- 3 Let ABC be a triangle and let I be the incenter. Let N, M be the midpoints of the sides AB and CA respectively. The lines BI and CI meet MN at K and L respectively. Prove that $AI + BI + CI > BC + KL$.

Greece

- 4 Determine the triangle with sides a, b, c and circumradius R for which $R(b + c) = a\sqrt{bc}$.

Romania

- 5 Let $n_1, n_2, \dots, n_{1998}$ be positive integers such that

$$n_1^2 + n_2^2 + \dots + n_{1997}^2 = n_{1998}^2.$$

Show that at least two of the numbers are even.
