Junior Balkan MO 1999

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1 Let a, b, c, x, y be five real numbers such that $a^3 + ax + y = 0$, $b^3 + bx + y = 0$ and $c^3 + cx + y = 0$. If a, b, c are all distinct numbers prove that their sum is zero.

Ciprus

2 For each nonnegative integer n we define $A_n = 2^{3n} + 3^{6n+2} + 5^{6n+2}$. Find the greatest common divisor of the numbers $A_0, A_1, \ldots, A_{1999}$.

Romania

3 Let S be a square with the side length 20 and let M be the set of points formed with the vertices of S and another 1999 points lying inside S. Prove that there exists a triangle with vertices in M and with area at most equal with $\frac{1}{10}$.

Yugoslavia

4 Let ABC be a triangle with $AB = AC$. Also, let $D \in [BC]$ be a point such that $BC > BD > DC > 0$, and let $\mathcal{C}_1, \mathcal{C}_2$ be the circumcircles of the triangles ABD and ADC respectively. Let BB' and CC' be diameters in the two circles, and let M be the midpoint of $B'C'$. Prove that the area of the triangle MBC is constant (i.e. it does not depend on the choice of the point D).

Greece