Junior Balkan MO 2000

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– June 23rd

1 Let x and y be positive reals such that

$$x^3 + y^3 + (x + y)^3 + 30xy = 2000.$$

Show that $x + y = 10$.

2 Find all positive integers $n \geq 1$ such that $n^2 + 3^n$ is the square of an integer.

Bulgaria

3 A half-circle of diameter EF is placed on the side BC of a triangle ABC and it is tangent to the sides AB and AC in the points Q and P respectively. Prove that the intersection point K between the lines EP and FQ lies on the altitude from A of the triangle ABC.

Albania

4 At a tennis tournament there were $2n$ boys and n girls participating. Every player played every other player. The boys won $\frac{7}{5}$ times as many matches as the girls. It is knowns that there were no draws. Find n.

Serbia