Junior Balkan MO 2001

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1	Solve the equation $a^3 + b^3 + c^3 = 2001$ in positive integers. *Mircea Becheanu, Romania*
2	Let ABC be a triangle with $\angle C = 90^\circ$ and $CA \neq CB$. Let CH be an altitude and CL be an interior angle bisector. Show that for $X \neq C$ on the line CL, we have $\angle XAC \neq \angle XBC$. Also show that for $Y \neq C$ on the line CH we have $\angle YAC \neq \angle YBC$. *Bulgaria*
3	Let ABC be an equilateral triangle and D, E points on the sides $[AB]$ and $[AC]$ respectively. If DF, EF (with $F \in AE$, $G \in AD$) are the interior angle bisectors of the angles of the triangle ADE, prove that the sum of the areas of the triangles DEF and DEG is at most equal with the area of the triangle ABC. When does the equality hold? *Greece*
4	Let N be a convex polygon with 1415 vertices and perimeter 2001. Prove that we can find 3 vertices of N which form a triangle of area smaller than 1.