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1 The triangle ABC has $CA = CB$. P is a point on the circumcircle between A and B (and on the opposite side of the line AB to C). D is the foot of the perpendicular from C to PB. Show that $PA + PB = 2 \cdot PD$.

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2 Two circles with centers O_1 and O_2 meet at two points A and B such that the centers of the circles are on opposite sides of the line AB. The lines BO_1 and BO_2 meet their respective circles again at B_1 and B_2. Let M be the midpoint of B_1B_2. Let M_1, M_2 be points on the circles of centers O_1 and O_2 respectively, such that $\angle AO_1M_1 = \angle AO_2M_2$, and B_1 lies on the minor arc AM_1 while B lies on the minor arc AM_2. Show that $\angle MM_1B = \angle MM_2B$.

Ciprus

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3 Find all positive integers which have exactly 16 positive divisors $1 = d_1 < d_2 < \ldots < d_{16} = n$ such that the divisor d_k, where $k = d_5$, equals $(d_2 + d_4)d_6$.

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4 Prove that for all positive real numbers a, b, c the following inequality takes place

$$\frac{1}{b(a+b)} + \frac{1}{c(b+c)} + \frac{1}{a(c+a)} \geq \frac{27}{2(a+b+c)^2}.$$

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