

**Junior Balkan MO 2004**

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1 Prove that the inequality

$$\frac{x+y}{x^2-xy+y^2} \leq \frac{2\sqrt{2}}{\sqrt{x^2+y^2}}$$

holds for all real numbers  $x$  and  $y$ , not both equal to 0.

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2 Let  $ABC$  be an isosceles triangle with  $AC = BC$ , let  $M$  be the midpoint of its side  $AC$ , and let  $Z$  be the line through  $C$  perpendicular to  $AB$ . The circle through the points  $B$ ,  $C$ , and  $M$  intersects the line  $Z$  at the points  $C$  and  $Q$ . Find the radius of the circumcircle of the triangle  $ABC$  in terms of  $m = CQ$ .

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3 If the positive integers  $x$  and  $y$  are such that  $3x + 4y$  and  $4x + 3y$  are both perfect squares, prove that both  $x$  and  $y$  are both divisible with 7.

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4 Consider a convex polygon having  $n$  vertices,  $n \geq 4$ . We arbitrarily decompose the polygon into triangles having all the vertices among the vertices of the polygon, such that no two of the triangles have interior points in common. We paint in black the triangles that have two sides that are also sides of the polygon, in red if only one side of the triangle is also a side of the polygon and in white those triangles that have no sides that are sides of the polygon.

Prove that there are two more black triangles than white ones.

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