Junior Balkan MO 2005

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1	Find all positive integers x, y satisfying the equation $$9(x^2 + y^2 + 1) + 2(3xy + 2) = 2005.$$
2	Let ABC be an acute-angled triangle inscribed in a circle k. It is given that the tangent from A to the circle meets the line BC at point P. Let M be the midpoint of the line segment AP and R be the second intersection point of the circle k with the line BM. The line PR meets again the circle k at point S different from R. Prove that the lines AP and CS are parallel.
3	Prove that there exist (a) 5 points in the plane so that among all the triangles with vertices among these points there are 8 right-angled ones; (b) 64 points in the plane so that among all the triangles with vertices among these points there are at least 2005 right-angled ones.
4	Find all 3-digit positive integers $\overline{abc}$ such that $$\overline{abc} = abc(a + b + c),$$ where $\overline{abc}$ is the decimal representation of the number.