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- 1 If $n > 4$ is a composite number, then $2n$ divides $(n - 1)!$.

- 2 The triangle ABC is isosceles with $AB = AC$, and $\angle BAC < 60^\circ$. The points D and E are chosen on the side AC such that, $EB = ED$, and $\angle ABD \equiv \angle CBE$. Denote by O the intersection point between the internal bisectors of the angles $\angle BDC$ and $\angle ACB$. Compute $\angle COD$.

- 3 We call a number *perfect* if the sum of its positive integer divisors (including 1 and n) equals $2n$. Determine all *perfect* numbers n for which $n - 1$ and $n + 1$ are prime numbers.

- 4 Consider a $2n \times 2n$ board. From the i th line we remove the central $2(i - 1)$ unit squares. What is the maximal number of rectangles 2×1 and 1×2 that can be placed on the obtained figure without overlapping or getting outside the board?
