

**Junior Balkan MO 2009**

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by delegat

- 1 Let  $ABCDE$  be a convex pentagon such that  $AB + CD = BC + DE$  and  $k$  a circle with center on side  $AE$  that touches the sides  $AB, BC, CD$  and  $DE$  at points  $P, Q, R$  and  $S$  (different from vertices of the pentagon) respectively. Prove that lines  $PS$  and  $AE$  are parallel.

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- 2 Solve in non-negative integers the equation  $2^a 3^b + 9 = c^2$

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- 3 Let  $x, y, z$  be real numbers such that  $0 < x, y, z < 1$  and  $xyz = (1-x)(1-y)(1-z)$ . Show that at least one of the numbers  $(1-x)y, (1-y)z, (1-z)x$  is greater than or equal to  $\frac{1}{4}$

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- 4 Each one of 2009 distinct points in the plane is coloured in blue or red, so that on every blue-centered unit circle there are exactly two red points. Find the greatest possible number of blue points.

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