Junior Balkan MO 2012

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–	June 27th
1	Let a, b, c be positive real numbers such that $a+b+c=1$. Prove that $$\frac{a}{b}+\frac{a}{c}+\frac{c}{b}+\frac{c}{a}+\frac{b}{c}+\frac{b}{a}+6 \geq 2\sqrt{2}\left(\sqrt{\frac{1-a}{a}}+\sqrt{\frac{1-b}{b}}+\sqrt{\frac{1-c}{c}}\right).$$ When does equality hold?
2	Let the circles k_1 and k_2 intersect at two points A and B, and let t be a common tangent of k_1 and k_2 that touches k_1 and k_2 at M and N respectively. If $t \perp AM$ and $MN = 2AM$, evaluate the angle NMB.
3	On a board there are n nails, each two connected by a rope. Each rope is colored in one of n given distinct colors. For each three distinct colors, there exist three nails connected with ropes of these three colors. a) Can n be 6 ? b) Can n be 7 ?
4	Find all positive integers x, y, z and t such that $2^x 3^y + 5^z = 7^t$.