Czech And Slovak Mathematical Olympiad, Round III, Category A 2004

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by littletush

1 Find all triples (x,y,z) of real numbers such that

$$x^2+y^2+z^2\le 6+\min(x^2-\frac{8}{x^4},y^2-\frac{8}{y^4},z^2-\frac{8}{z^4}).$$

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2 Consider all words containing only letters A and B. For any positive integer n, $p(n)$ denotes the number of all n-letter words without four consecutive A's or three consecutive B's. Find the value of the expression

$$\frac{p(2004)-p(2002)-p(1999)}{p(2001)+p(2000)}.$$

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3 Given a circle S and its 121 chords $P_i(i=1,2,\ldots,121)$, each with a point $A_i(i=1,2,\ldots,121)$ on it. Prove that there exists a point X on the circumference of S such that: there exist 29 distinct indices $1\le k_1\le k_2\le\ldots\le k_{29}\le 121$, such that the angle formed by $A_{k_j}X$ and P_{k_j} is smaller than 21 degrees for every $j=1,2,\ldots,29$.

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4 Find all positive integers n such that $\sum_{k=1}^{n}\frac{n}{k!}$ is an integer.

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5 Let L be an arbitrary point on the minor arc CD of the circumcircle of square $ABCD$. Let K,M,N be the intersection points of AL,CD; CL,AD; and MK,BC respectively. Prove that B,M,L,N are concyclic.

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6 Find all functions $f:\mathbb{R}^+\to\mathbb{R}^+$ such that for all positive real numbers x,y,

$$x^2[f(x)+f(y)]=(x+y)f(yf(x)).$$

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