Israel Team Selection Test 2016

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–	**TST 1**
1	Let a, b, c be positive numbers satisfying $ab + bc + ca + 2abc = 1$. Prove that $4a + b + c \geq 2$.
2	Rothschild the benefactor has a certain number of coins. A man comes, and Rothschild wants to share his coins with him. If he has an even number of coins, he gives half of them to the man and goes away. If he has an odd number of coins, he donates one coin to charity so he can have an even number of coins, but meanwhile another man comes. So now he has to share his coins with two other people. If it is possible to do so evenly, he does so and goes away. Otherwise, he again donates a few coins to charity (no more than 3). Meanwhile, yet another man comes. This goes on until Rothschild is able to divide his coins evenly or until he runs out of money. Does there exist a natural number N such that if Rothschild has at least N coins in the beginning, he will end with at least one coin?
3	Prove that there exists an ellipsoid touching all edges of an octahedron if and only if the octahedron's diagonals intersect. (Here an octahedron is a polyhedron consisting of eight triangular faces, twelve edges, and six vertices such that four faces meat at each vertex. The diagonals of an octahedron are the lines connecting pairs of vertices not connected by an edge).
4	A regular 60-gon is given. What is the maximum size of a subset of its vertices containing no isosceles triangles?
–	**TST 2**
1	A square $ABCD$ is given. A point P is chosen inside the triangle ABC such that $\angle CAP = 15^\circ = \angle BCP$. A point Q is chosen such that $APCQ$ is an isosceles trapezoid: $PC \parallel AQ$, and $AP = CQ, AP \nparallel CQ$. Denote by N the midpoint of PQ. Find the angles of the triangle CAN.
2	Find all $f : \mathbb{R} \to \mathbb{R}$ satisfying (for all $x, y \in \mathbb{R}$): $f(x+y)^2 - f(2x^2) = f(y-x)f(y+x) + 2x \cdot f(y)$.
3	On each square of an nxn board sleeps a dragon. Two dragons are called neighbors if their squares have a side in common. Each turn, Minnie wakes up a dragon which has a living neighbor and Max directs it towards one of its living neighbors. The dragon than breathes fire on that neighbor and destroys it, and then goes back to sleep. Minnie's goal is to minimize the snoring of the dragons and leave as few living dragons as possible. Max is a member of PETD (People for the Ethical Treatment of Dragons), and he

wants to save as many dragons as he can.

How many dragons will stay alive at the end if

1. $n=4$?
2. $n=5$?

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4 Find the greatest common divisor of all numbers of the form $(2^{a^2}\cdot 19^{b^2}\cdot 53^{c^2}+8)^{16}-1$ where a,b,c are integers.
