Lusophon Mathematical Olympiad 2011

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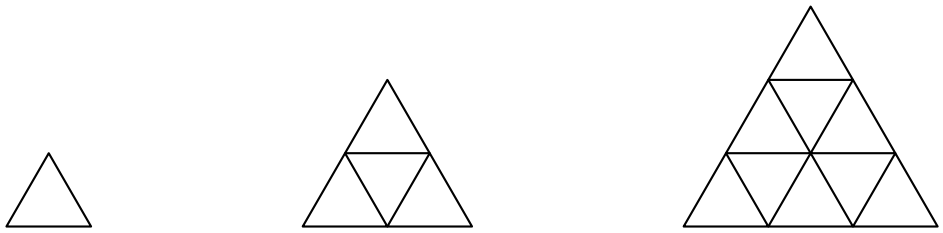
Day 1

1 Prove that the area of the circle inscribed in a regular hexagon is greater than 90% of the area of the hexagon.

2 A non-negative integer n is said to be *squaredigital* if it equals the square of the sum of its digits. Find all non-negative integers which are squaredigital.

3 Consider a sequence of equilateral triangles T_n as represented below:



The length of the side of the smallest triangles is 1. A triangle is called a delta if its vertex is at the top; for example, there are 10 deltas in T_3. A delta is said to be perfect if the length of its side is even. How many perfect deltas are there in T_{20}?

Day 2

1 Each one of three friends, Mrio, Joo and Filipe, does one, and only one, of the following sports: football, basketball and swimming. None of these sports is done by more than one of the friends. Each one of the friends likes a certain kind of fruit: one likes oranges, another likes bananas and the other likes papayas. Find, for each one, which sport he plays and which fruit he prefers, given that:

* Mrio doesn't like oranges;

* Joo doesn't play football;

* The swimmer hates bananas;

* The swimmer and the one who likes oranges do different sports;

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* The one who likes papayas and the footballer visit Filipe every Saturday.

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2 Consider two circles, tangent at T, both inscribed in a rectangle of height 2 and width 4. A point E moves counterclockwise around the circle on the left, and a point D moves clockwise around the circle on the right. E and D start moving at the same time; E starts at T, and D starts at A, where A is the point where the circle on the right intersects the top side of the rectangle. Both points move with the same speed. Find the locus of the midpoints of the segments joining E and D.

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3 Let d be a positive real number. The scorpion tries to catch the flea on a 10×10 chessboard. The length of the side of each small square of the chessboard is 1. In this game, the flea and the scorpion move alternately. The flea is always on one of the 121 vertexes of the chessboard and, in each turn, can jump from the vertex where it is to one of the adjacent vertexes. The scorpion moves on the boundary line of the chessboard, and, in each turn, it can walk along any path of length less than d.
At the beginning, the flea is at the center of the chessboard and the scorpion is at a point that he chooses on the boundary line. The flea is the first one to play. The flea is said to *escape* if it reaches a point of the boundary line, which the scorpion can't reach in the next turn. Obviously, for big values of d, the scorpion has a strategy to prevent the flea's escape. For what values of d can the flea escape? Justify your answer.

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