

Lusophon Mathematical Olympiad 2013

www.artofproblemsolving.com/community/c4237

by DavidAndrade

- 1 If Xiluva puts two oranges in each basket, four oranges are in excess. If she puts five oranges in each basket, one basket is in excess. How many oranges and baskets has Xiluva?

- 2 Let ABC be an acute triangle. The circumference with diameter AB intersects sides AC and BC at E and F respectively. The tangent lines to the circumference at the points E and F meet at P . Show that P belongs to the altitude from C of triangle ABC .

- 3 An event occurs many years ago. It occurs periodically in x consecutive years, then there is a break of y consecutive years. We know that the event occurred in 1964, 1986, 1996, 2008 and it didn't occur in 1976, 1993, 2006, 2013. What is the first year in that the event will occur again?

- 4 Find all the pairs (x, y) of positive integers that satisfy the equation $x^2 - xy + 2x - 3y = 2013$.

- 5 Find all the numbers of 5 non-zero digits such that deleting consecutively the digit of the left, in each step, we obtain a divisor of the previous number.

- 6 Consider a triangle ABC . Let S be a circumference in the interior of the triangle that is tangent to the sides BC, CA, AB at the points D, E, F respectively. In the exterior of the triangle we draw three circumferences S_A, S_B, S_C . The circumference S_A is tangent to BC at L and to the prolongation of the lines AB, AC at the points M, N respectively. The circumference S_B is tangent to AC at E and to the prolongation of the line BC at P . The circumference S_C is tangent to AB at F and to the prolongation of the line BC at Q . Show that the lines EP, FQ and AL meet at a point of the circumference S .