Finnish National High School Mathematics Competition 1997

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by socrates

1   Determine the real numbers a such that the equation $a3^x + 3^{-x} = 3$ has exactly one solution x.

2   Circles with radii R and r $(R > r)$ are externally tangent. Another common tangent of the circles in drawn.
This tangent and the circles bound a region inside which a circle as large as possible is drawn. What is the radius of this circle?

3 12 knights are sitting at a round table. Every knight is an enemy with two of the adjacent knights but with none of the others. 5 knights are to be chosen to save the princess, with no enemies in the group. How many ways are there for the choice?

4   Count the sum of the four-digit positive integers containing only odd digits in their decimal representation.

5   For an integer $n \geq 3$, place n points on the plane in such a way that all the distances between the points are at most one and exactly n of the pairs of points have the distance one.