Finnish National High School Mathematics Competition 1999

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by socrates

1 Show that the equation $x^3 + 2y^2 + 4z = n$ has an integral solution (x, y, z) for all integers n.

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2 Suppose that the positive numbers $a_1, a_2, .., a_n$ form an arithmetic progression; hence $a_{k+1} - a_k = d$, for $k = 1, 2, ..., n - 1$.
Prove that
$$\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_{n-1}a_n} = \frac{n-1}{a_1a_n}.$$

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3 Determine how many primes are there in the sequence
$$101, 10101, 1010101....$$

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4 Three unit circles have a common point O. The other points of (pairwise) intersection are A, B and C. Show that the points A, B and C are located on some unit circle.

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5 An ordinary domino tile can be identifi ed as a pair (k, m) where numbers k and m can get values $0, 1, 2, 3, 4, 5$ and 6.
Pairs (k, m) and (m, k) determine the same tile. In particular, the pair (k, k) determines one tile.
We say that two domino tiles *match*, if they have a common component.
Generalized n-domino tiles m and k can get values $0, 1, ..., n$.
What is the probability that two randomly chosen n-domino tiles match?
