Finnish National High School Mathematics Competition 2000

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by socrates

1 Two circles are externally tangent at the point A. A common tangent of the circles meets one circle at the point B and another at the point C ($B \neq C$). Line segments BD and CE are diameters of the circles. Prove that the points D, A and C are collinear.

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2 Prove that the integral part of the decimal representation of the number $(3+\sqrt{5})^n$ is odd, for every positive integer n.

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3 Determine the positive integers n such that the inequality

$$n! > \sqrt{n^n}$$

holds.

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4 There are seven points on the plane, no three of which are collinear. Every pair of points is connected with a line segment, each of which is either blue or red. Prove that there are at least four monochromatic triangles in the fi gure.

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5 Irja and Valtteri are tossing coins. They take turns, Irja starting. Each of them has a pebble which reside on opposite vertices of a square at the start. If a player gets heads, she or he moves her or his pebble on opposite vertex. Otherwise the player in turn moves her or his pebble to an adjacent vertex so that Irja proceeds in positive and Valtteri in negative direction. The winner is the one who can move his pebble to the vertex where opponent's pebble lies. What is the probability that Irja wins the game?
