Finnish National High School Mathematics Competition 2001

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by socrates

1 In the right triangle ABC, CF is the altitude based on the hypotenuse AB.
The circle centered at B and passing through F and the circle with centre A and the same radius intersect at a point of CB.
Determine the ratio $FB : BC$.

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2 Equations of non-intersecting curves are $y = ax^2 + bx + c$ and $y = dx^2 + ex + f$ where $ad < 0$.
Prove that there is a line of the plane which does not meet either of the curves.

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3 Numbers a, b and c are positive integers and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} < 1$. Show that

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \leq \frac{41}{42}.$$

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4 A sequence of seven digits is randomly chosen in a weekly lottery. Every digit can be any of the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$.
What is the probability of having at most fi ve diff erent digits in the sequence?

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5 Determine $n \in \mathbb{N}$ such that $n^2 + 2$ divides $2 + 2001n$.
