Finnish National High School Mathematics Competition 2004

www.artofproblemsolving.com/community/c4251

by socrates

1 The equations $x^2+2ax+b^2=0$ and $x^2+2bx+c^2=0$ both have two diff erent real roots.
Determine the number of real roots of the equation $x^2+2cx+a^2=0$.

---

2 a, b and c are positive integers and

$$\frac{a\sqrt{3}+b}{b\sqrt{3}+c}$$

is a rational number.
Show that

$$\frac{a^2+b^2+c^2}{a+b+c}$$

is an integer.

---

3 Two circles with radii r and R are externally tangent.
Determine the length of the segment cut from the common tangent of the circles by the other common tangents.

---

4 The numbers $2005!+2, 2005!+3, ..., 2005!+2005$ form a sequence of 2004 consequtive integers, none of which is a prime number.
Does there exist a sequence of 2004 consequtive integers containing exactly 12 prime numbers?

---

5 Finland is going to change the monetary system again and replace the Euro by the Finnish Mark.
The Mark is divided into 100 pennies.
There shall be coins of three denominations only, and the number of coins a person has to carry in order to be able
to pay for any purchase less than one mark should be minimal.
Determine the coin denominations.
