

AoPS Community 2005 Finnish National High School Mathematics Competition

Finnish National High School Mathematics Competition 2005

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by socrates

- 1 In the figure below, the centres of four squares have been connected by two line segments. Prove that these line segments are perpendicular.

- 2 There are 12 seats at a round table in a restaurant. A group of five women and seven men arrives at the table. How many ways are there for choosing the sitting order, provided that every woman ought to be surrounded by two men and two orders are regarded as different, if at least one person has a different neighbour on one's right side.

- 3 Solve the group of equations:
$$\begin{cases} (x + y)^3 = z \\ (y + z)^3 = x \\ (z + x)^3 = y \end{cases}$$

- 4 The numbers 1, 3, 7 and 9 occur in the decimal representation of an integer. Show that permuting the order of digits one can obtain an integer divisible by 7.

- 5 A finite sequence is said to be *disorderly*, if no two terms of the sequence have their average in between them. For example, $(0, 2, 1)$ is disorderly, for $1 = \frac{0+2}{2}$ is not in between 0 and 2, and the other averages $\frac{0+1}{2} = \frac{1}{2}$ and $\frac{2+1}{2} = 1\frac{1}{2}$ do not even occur in the sequence. Prove that for every $n \in \mathbb{N}$ there is a disorderly sequence enumerating the numbers $0, 1, \dots, n$ without repetitions.