

**Finnish National High School Mathematics Competition 2007**

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by socrates

- 1 Show: when a prime number is divided by 30, the remainder is either 1 or a prime number. Is a similar claim true, when the divisor is 60 or 90?
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- 2 Determine the number of real roots of the equation

$$x^8 - x^7 + 2x^6 - 2x^5 + 3x^4 - 3x^3 + 4x^2 - 4x + \frac{5}{2} = 0.$$

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- 3 There are five points in the plane, no three of which are collinear. Show that some four of these points are the vertices of a convex quadrilateral.
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- 4 The six offices of the city of Salavaara are to be connected to each other by a communication network which utilizes modern picotechnology. Each of the offices is to be connected to all the other ones by direct cable connections. Three operators compete to build the connections, and there is a separate competition for every connection.

When the network is finished one notices that the worst has happened: the systems of the three operators are incompatible. So the city must reject connections built by two of the operators, and these are to be chosen so that the damage is minimized. What is the minimal number of offices which still can be connected to each other, possibly through intermediate offices, in the worst possible case.

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- 5 Show that there exists a polynomial  $P(x)$  with integer coefficients, such that the equation  $P(x) = 0$  has no integer solutions, but for each positive integer  $n$  there is an  $x \in \mathbb{Z}$  such that  $n \mid P(x)$ .
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