Finnish National High School Mathematics Competition 2008

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by socrates

1   Foxes, wolves and bears arranged a big rabbit hunt. There were 45 hunters catching 2008 rabbits.
Every fox caught 59 rabbits, every wolf 41 rabbits and every bear 40 rabbits.
How many foxes, wolves and bears were there in the hunting company?

2   The incentre of the triangle ABC is I. The lines AI, BI and CI meet the circumcircle of the triangle ABC also at points D, E and F, respectively.
Prove that AD and EF are perpendicular.

3   Solve the diophantine equation
$$x^{2008} - y^{2008} = 2^{2009}.$$

4   Eight football teams play matches against each other in such a way that no two teams meet twice and no three teams play all of the three possible matches.
What is the largest possible number of matches?

5   The closed line segment I is covered by finitely many closed line segments.
Show that one can choose a subfamily S of the family of line segments having the properties:
(1) the chosen line segments are disjoint,
(2) the sum of the lengths of the line segments of S is more than half of the length of I.

Show that the claim does not hold any more if the line segment I is replaced by a circle and other occurences of the compound word "line segment" by the word "circular arc".