Finnish National High School Mathematics Competition 2010

www.artofproblemsolving.com/community/c4257

by socrates

1 Let ABC be right angled triangle with sides s_1, s_2, s_3 medians m_1, m_2, m_3. Prove that $m_1^2+m_2^2+m_3^2=\frac{3}{4}(s_1^2+s_2^2+s_3^2)$.

---

2 Determine the least $n \in \mathbb{N}$ such that $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$ has at least 2010 positive factors.

---

3 Let $P(x)$ be a polynomial with integer coefficients and roots 1997 and 2010. Suppose further that $|P(2005)| < 10$. Determine what integer values $P(2005)$ can get.

---

4 In a football season, even number n of teams plays a simple series, i.e. each team plays once against each other team. Show that ona can group the series into $n-1$ rounds such that in every round every team plays exactly one match.

---

5 Let S be a non-empty subset of a plane. We say that the point P can be seen from A if every point from the line segment AP belongs to S. Further, the set S can be seen from A if every point of S can be seen from A. Suppose that S can be seen from A, B and C where ABC is a triangle. Prove that S can also be seen from any other point of the triangle ABC.
