Finnish National High School Mathematics Competition 2011

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1   An equilateral triangle has been drawn inside the circle. Split the triangle to two parts with equal area by a line segment parallel to the triangle side. Draw an inscribed circle inside this smaller triangle. What is the ratio of the area of this circle compared to the area of original circle.

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2   Find all integers x and y satisfying the inequality

$$x^4 - 12x^2 + x^2y^2 + 30 \le 0.$$

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3   Points D and E divides the base BC of an isosceles triangle ABC into three equal parts and D is between B and E. Show that $\angle BAD < \angle DAE$.

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4   Show that there is a perfect square (a number which is a square of an integer) such that sum of its digits is 2011.

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5   Two players, the builder and the destroyer, plays the following game. Builder starts and players chooses alternatively different elements from the set $\{0, 1, \ldots, 10\}$. Builder wins if some four integer of those six integer he chose forms an arithmetic sequence. Destroyer wins if he can prevent to form such an arithmetic four-tuple. Which one has a winning strategy?