**Finnish National High School Mathematics Competition 2012**

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**1** A secant line splits a circle into two segments. Inside those segments, one draws two squares such that both squares has two corners on a secant line and two on the circumference. The ratio of the square's side lengths is $5:9$. Compute the ratio of the secant line versus circle radius.

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**2** Let $x\neq 1, y\neq 1$ and $x\neq y$. Show that if

$$\frac{yz-x^2}{1-x}=\frac{zx-y^2}{1-y},$$

then

$$\frac{yz-x^2}{1-x}=\frac{zx-y^2}{1-y}=x+y+z.$$

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**3** Prove that for all integers $k\geq 2$, the number $k^{k-1}-1$ is divisible by $(k-1)^2$.

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**4** Let $k,n\in\mathbb{N}, 0<k<n$. Prove that

$$\sum_{j=1}^{k}\binom{n}{j}=\binom{n}{1}+\binom{n}{2}+\ldots+\binom{n}{k}\leq n^k.$$

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**5** The [i]Collatz's function[i] is a mapping $f:\mathbb{Z}_+\to\mathbb{Z}_+$ satisfying

$$f(x)=\begin{cases} 3x+1, & \text{as } x \text{ is odd} \\ x/2, & \text{as } x \text{ is even.}\end{cases}$$

In addition, let us define the notation $f^1=f$ and inductively $f^{k+1}=f\circ f^k$, or to say in another words, $f^k(x)=\underbrace{f(\ldots(f}_{k\text{ times}}(x)\ldots).$

Prove that there is an $x\in\mathbb{Z}_+$ satisfying

$$f^{40}(x)>2012x.$$

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