

Finnish National High School Mathematics Competition 2013

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1 The coefficients a, b, c of a polynomial $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^3 + ax^2 + bx + c$ are mutually distinct integers and different from zero. Furthermore, $f(a) = a^3$ and $f(b) = b^3$. Determine a, b and c .

2 In a particular European city, there are only 7 day tickets and 30 day tickets to the public transport. The former costs 7.03 euro and the latter costs 30 euro. Aina the Algebraist decides to buy at once those tickets that she can travel by the public transport the whole three year (2014-2016, 1096 days) visiting in the city. What is the cheapest solution?

3 The points A, B , and C lies on the circumference of the unit circle. Furthermore, it is known that AB is a diameter of the circle and

$$\frac{|AC|}{|CB|} = \frac{3}{4}.$$

The bisector of ABC intersects the circumference at the point D . Determine the length of the AD .

4 A subset E of the set $\{1, 2, 3, \dots, 50\}$ is said to be *special* if it does not contain any pair of the form $\{x, 3x\}$. A special set E is *superspecial* if it contains as many elements as possible. How many element there are in a superspecial set and how many superspecial sets there are?

5 Find all integer triples (m, p, q) satisfying

$$2^m p^2 + 1 = q^5$$

where $m > 0$ and both p and q are prime numbers.
