Kosovo Team Selection Test 2017

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by Duarti

1 Find all positive integers (a,b), such that $\frac{a^2}{2ab^2-b^3+1}$ is also a positive integer.

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2 Prove that there doesn't exist any function $f:\mathbb{N}\to\mathbb{N}$ such that : $f(f(n-1)=f(n+1)-f(n)$, for every natural $n\geq 2$

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3 If a and b are positive real numbers with sum 3, and x,y,z positive real numbers with product 1, prove that : $(ax+b)(ay+b)(az+b)\geq 27$

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4 For every $n\in\mathbb{N}_0$, prove that $\sum_{k=0}^{[\frac{n}{2}]} 2^{n-2k}\binom{n}{2k}=\frac{3^n+1}{2}$

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5 Given triangle ABC. Let P, Q, R, be the tangency points of inscribed circle of $\triangle ABC$ on sides AB, BC, AC respectively. We take the reflection of these points with respect to midpoints of the sides they lie on, and denote them as P',Q' and R'. Prove that AP', BQ', and CR' are concurrent.
