Uzbekistan National Olympiad 2005

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1 Given a,b c are lenth of a triangle (If ABC is a triangle then AC=b, BC=a, AC=b) and $a+b+c=2$. Prove that $1+abc<ab+bc+ca\leq\frac{28}{27}+abc$

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2 Solve in integer the equation $\frac{1}{2}(x+y)(y+z)(x+z)+(x+y+z)^3=1-xyz$

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3 Find the last five digits of $1^{100}+2^{100}+3^{100}+...+999999^{100}$

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4 Let $ABCD$ is a cyclic. K,L,M,N are midpoints of segments AB, BC CD and DA. H_1,H_2,H_3,H_4 are orthocenters of AKN KBL LCM and MND. Prove that $H_1H_2H_3H_4$ is a paralelogram.
