Uzbekistan National Olympiad 2011

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Day 1

1 Let a,b,c Postive real numbers such that $a+b+c \geq 6$. Find the minimum value $A = \sum_{cyc} a^2$+$\sum_{cyc} \frac{a}{b^2+c+1}$

2 Prove that $\forall n \in \mathbb{N}$,$\exists a, b, c \in$$\bigcup_{k\in\mathbb{N}}(k^2, k^2 + k + 3\sqrt{3})$ such that $n = \frac{ab}{c}$.

3 Given an acute triangle ABC with altituties AD and BE. O circumcinter of ABC.If o lies on the segment DE then find the value of $sinAsinBcosC$

4 A graph G arises from G_1 and G_2 by pasting them along S if G has induced subgraphs G_1, G_2 with $G = G_1 \cup G_2$ and S is such that $S = G_1 \cap G_2$. A is graph is called *chordal* if it can be constructed recursively by pasting along complete subgraphs, starting from complete subgraphs. For a graph $G(V, E)$ define its Hilbert polynomial $H_G(x)$ to be $H_G(x) = 1+Vx+Ex^2+c(K_3)x^3 + c(K_4)x^4 + \ldots + c(K_{w(G)})x^{w(G)}$,
where $c(K_i)$ is the number of i-cliques in G and $w(G)$ is the clique number of G. Prove that $H_G(-1) = 0$ if and only if G is chordal or a tree.

Day 2

1 Find the minimum value of $|x - y| + \sqrt{(x + 2)^2 + (y - 4)^4}$

2 Let triangle ABC with $AB = c$ $AC = b$ $BC = a$ R circumradius, p half peremetr of ABC.
I f $\frac{acosA+bcosB+ccosC}{asinA+bsinB+csinC} = \frac{p}{9R}$ then find all value of $cosA$.

3 In acute triangle ABC AD is bisector. O is circumcenter, H is orthocenter. If $AD = AC$ and $AC \perp OH$. Find all of the value of $\angle ABC$ and $\angle ACB$.

4 Does existes a function $f : N- > N$ and for all positeve integer n $f(f(n) + 2011) = f(n) + f(f(n))$