Uzbekistan National Olympiad 2012

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1 Given a digits $0, 1, 2, ..., 9$. Find the number of numbers of 6 digits which cantain 7 or 7's digit and they is permulated(For example 137456 and 314756 is one numbers).

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2 For any positive integers n and m satisfying the equation $n^3+(n+1)^3+(n+2)^3=m^3$, prove that $4 \mid n+1$.

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3 The inscribed circle ω of the non-isosceles acute-angled triangle ABC touches the side BC at the point D. Suppose that I and O are the centres of inscribed circle and circumcircle of triangle ABC respectively. The circumcircle of triangle ADI intersects AO at the points A and E. Prove that AE is equal to the radius r of ω.

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4 Given a, b and c positive real numbers with $ab+bc+ca=1$. Then prove that $\frac{a^3}{1+9b^2ac}+\frac{b^3}{1+9c^2ab}+\frac{c^3}{1+9a^2bc}\geq\frac{(a+b+c)^3}{18}$

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5 Given points A, B, C and D lie a circle. $AC \cap BD = K$. I_1, I_2, I_3 and I_4 incenters of ABK, BCK, CDK, DKA M_1, M_2, M_3, M_4 midpoints of arcs AB, BC, CA, DA . Then prove that $M_1I_1, M_2I_2, M_3I_3, M_4I_4$ are concurrent.
