Second Round Olympiad 2002

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by tau172

1 In $\triangle ABC$, $\angle A=60$, $AB>AC$, point O is the circumcenter and H is the intersection point of two altitudes BE and CF. Points M and N are on the line segments BH and HF respectively, and satisfy $BM=CN$. Determine the value of $\frac{MH+NH}{OH}$.

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2 There are real numbers a,b and c and a positive number λ such that $f(x)=x^3+ax^2+bx+c$ has three real roots x_1,x_2 and x_3 satisfying $(1)x_2-x_1=\lambda$ $(2)x_3>\frac{1}{2}(x_1+x_2)$.
Find the maximum value of $\frac{2a^3+27c-9ab}{\lambda^3}$

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3 Before The World Cup tournament, the football coach of F country will let seven players, $A_1,A_2,\dots,A_7$, join three training matches (90 minutes each) in order to assess them. Suppose, at any moment during a match, one and only one of them enters the field, and the total time (which is measured in minutes) on the field for each one of A_1,A_2,A_3, and A_4 is divisible by 7 and the total time for each of A_5,A_6, and A_7 is divisible by 13. If there is no restriction about the number of substitutions of players during each match, then how many possible cases are there within the total time for every player on the field?
