

Second Round Olympiad 2003

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by tau172

- 1 From point P outside a circle draw two tangents to the circle touching at A and B . Draw a secant line intersecting the circle at points C and D , with C between P and D . Choose point Q on the chord CD such that $\angle DAQ = \angle PBC$. Prove that $\angle DBQ = \angle PAC$.

- 2 Let the three sides of a triangle be ℓ, m, n , respectively, satisfying $\ell > m > n$ and $\left\{\frac{3^\ell}{10^4}\right\} = \left\{\frac{3^m}{10^4}\right\} = \left\{\frac{3^n}{10^4}\right\}$, where $\{x\} = x - [x]$ and $[x]$ denotes the integral part of the number x . Find the minimum perimeter of such a triangle.

- 3 Let a space figure consist of n vertices and l lines connecting these vertices, with $n = q^2 + q + 1$, $l \geq q^2(q+1)^2 + 1$, $q \geq 2$, $q \in \mathbb{N}$. Suppose the figure satisfies the following conditions: every four vertices are non-coplanar, every vertex is connected by at least one line, and there is a vertex connected by at least $p+2$ lines. Prove that there exists a space quadrilateral in the figure, i.e. a quadrilateral with four vertices A, B, C, D and four lines AB, BC, CD, DA in the figure.