

Second Round Olympiad 2005

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by tau172

- 1 In $\triangle ABC$, $AB > AC$, l is a tangent line of the circumscribed circle of $\triangle ABC$, passing through A . The circle, centered at A with radius AC , intersects AB at D , and line l at E, F . Prove that lines DE, DF pass through the incenter and an excenter of $\triangle ABC$ respectively.

- 2 Assume that positive numbers a, b, c, x, y, z satisfy $cy + bz = a$, $az + cx = b$, and $bx + ay = c$. Find the minimum value of the function

$$f(x, y, z) = \frac{x^2}{x+1} + \frac{y^2}{y+1} + \frac{z^2}{z+1}.$$

- 3 For each positive integer, define a function

$$f(n) = \begin{cases} 0, & \text{if } n \text{ is the square of an integer} \\ \left\lfloor \frac{1}{\{\sqrt{n}\}} \right\rfloor, & \text{if } n \text{ is not the square of an integer} \end{cases}.$$

Find the value of $\sum_{k=1}^{200} f(k)$.