Second Round Olympiad 2007

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1 In an acute triangle ABC, $AB < AC$. AD is the altitude dropped onto BC and P is a point on AD. Let $PE \perp AC$ at E, $PF \perp AB$ at F and let J, K be the circumcentres of triangles BDF, CDE respectively. Prove that J, K, E, F are concyclic if and only if P is the orthocentre of triangle ABC.

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2 In a 7×8 chessboard, 56 stones are placed in the squares. Now we have to remove some of the stones such that after the operation, there are no five adjacent stones horizontally, vertically or diagonally. Find the minimal number of stones that have to be removed.

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3 For positive integers k, m, where $1 \le k \le 5$, define the function $f(m, k)$ as

$$f(m,k) = \sum_{i=1}^{5} \left[m\sqrt{\frac{k+1}{i+1}} \right]$$

where $[x]$ denotes the greatest integer not exceeding x. Prove that for any positive integer n, there exist positive integers k, m, where $1 \le k \le 5$, such that $f(m, k) = n$.
