Second Round Olympiad 2008

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1 Given a convex quadrilateral with $\angle B+\angle D<180$.Let P be an arbitrary point on the plane,define $f(P)=PA*BC+PD*CA+PC*AB$.
(1)Prove that P,A,B,C are concyclic when $f(P)$ attains its minimum.
(2)Suppose that E is a point on the minor arc AB of the circumcircle O of ABC,such that$AE=\frac{\sqrt{3}}{2}AB,BC=(\sqrt{3}-1)EC,\angle ECA=2\angle ECB$.Knowing that DA,DC are tangent to circle O,$AC=\sqrt{2}$,find the minimum of $f(P)$.

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2 Let $f(x)$ be a periodic function with periods T and $1(0<T<1)$.Prove that:
(1)If T is rational,then there exists a prime p such that $\frac{1}{p}$ is also a period of f;
(2)If T is irrational,then there exists a strictly decreasing infinite sequence a_n,with $1>a_n>0$ for all positive integer n,such that all a_n are periods of f.

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3 For all $k=1,2,\dots,2008$,$a_k>0$.Prove that iff $\sum_{k=1}^{2008}a_k>1$,there exists a function $f:N\rightarrow R$ satisfying
(1)$0=f(0)<f(1)<f(2)<\dots$;
(2)$f(n)$ has a finite limit when n approaches infinity;
(3)$f(n)-f(n-1)=\sum_{k=1}^{2008}a_kf(n+k)-\sum_{k=0}^{2007}a_{k+1}f(n+k)$,for all $n=1,2,3,\dots$.
