Second Round Olympiad 2010

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1 Given an acute triangle whose circumcenter is O.let K be a point on BC,different from its midpoint.D is on the extension of segment AK, BD and AC,CDandABintersect at N, M respectively.prove that A, B, D, C are concyclic.

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2 Given a fixed integer $k > 0, r = k + 0.5$,define $f^1(r) = f(r) = r[r], f^l(r) = f(f^{l-1}(r))(l > 1)$ where $[x]$ denotes the smallest integer not less than x.
prove that there exists integer m such that $f^m(r)$ is an integer.

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3 let $n > 2$ be a fixed integer.positive reals $a_i \leq 1$(for all $1 \leq i \leq n$).for all $k = 1, 2, ..., n$,let $A_k = \frac{\sum_{i=1}^{k} a_i}{k}$
prove that $|\sum_{k=1}^{n} a_k - \sum_{k=1}^{n} A_k| < \frac{n-1}{2}$.

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4 the code system of a new 'MO lock' is a regular n-gon,each vertex labelled a number 0 or 1 and coloured red or blue.it is known that for any two adjacent vertices,either their numbers or colours coincide.
find the number of all possible codes(in terms of n).
