Canada National Olympiad 2017

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by InCtrl

1 For pairwise distinct nonnegative reals a, b, c, prove that

$$\frac{a^2}{(b-c)^2} + \frac{b^2}{(c-a)^2} + \frac{c^2}{(b-a)^2} > 2$$

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2 Define a function $f(n)$ from the positive integers to the positive integers such that $f(f(n))$ is the number of positive integer divisors of n. Prove that if p is a prime, then $f(p)$ is prime.

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3 Define S_n as the set $1, 2, \cdots, n$. A non-empty subset T_n of S_n is called $balanced$ if the average of the elements of T_n is equal to the median of T_n. Prove that, for all n, the number of balanced subsets T_n is odd.

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4 Let $ABCD$ be a parallelogram. Points P and Q lie inside $ABCD$ such that $\triangle ABP$ and $\triangle BCQ$ are equilateral. Prove that the intersection of the line through P perpendicular to PD and the line through Q perpendicular to DQ lies on the altitude from B in $\triangle ABC$.

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5 There are 100 circles of radius one in the plane. A triangle formed by the centres of any three given circles has area at most 2017. Prove that there is a line intersecting at least three of the circles.
