

Puerto Rico Team Selection Test 2012

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- 1 Let x, y and z be consecutive integers such that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{1}{45}.$$

Find the maximum value of $x + y + z$.

- 2 A cone is constructed with a semicircular piece of paper, with radius 10. Find the height of the cone.
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- 3 ABC is a triangle that is inscribed in a circle. The angle bisectors of A, B, C meet the circle at D, E, F , respectively. Show that AD is perpendicular to EF .
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- 4 Let a, b, c, d be digits such that $d > c > b > a \geq 0$. How many numbers of the form $1a1b1c1d1$ are multiples of 33?
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- 5 A point P is outside of a circle and the distance to the center is 13. A secant line from P meets the circle at Q and R so that the exterior segment of the secant, PQ , is 9 and QR is 7. Find the radius of the circle.
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- 6 The increasing sequence 1; 3; 4; 9; 10; 12; 13; 27; 28; 30; 31, ... is formed with positive integers which are powers of 3 or sums of different powers of 3. Which number is in the 100th position?
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- 7 Let f be a function with the following properties:
- 1) $f(n)$ is defined for every positive integer n ;
 - 2) $f(n)$ is an integer;
 - 3) $f(2) = 2$;
 - 4) $f(mn) = f(m)f(n)$ for all m and n ;
 - 5) $f(m) > f(n)$ whenever $m > n$.

Prove that $f(n) = n$.
